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An Analysis of a queuing model with a departing server

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Abstract

In this overview, we briefly concentrate on variants of arrival and service processes with different sorts of working vacation models. The idea of working vacations has wide reach application in PC communication frameworks, manufacturing/production frameworks and inventory frameworks in particularly network service, web service, file move service and mail service and so on. The motivation of this work is to provide sufficient information to examiners, supervisors and industry individuals who are interested in using queuing hypothesis to demonstrate congestion issues and need to find the details of pertinent models.

Keywords: Server's vacation, Queueing model, Mathematical model

Introduction

Mass queuing frameworks are normal, in actuality, situations, for example, lifts, loading and unloading cargoes, giant wheel, chemical manufacturing process, communication organization, tourism, and so on. Cluster service queuing frameworks originated with [1]. [2] Have studied cluster service queuing frameworks. A detailed work on mass lines examined in [3]. The first concentrate in the variable server capacity mass service rule can be found. As of late many creators have considered queuing models with a variable server capacity mass service rule, for example, [4] [5]. Clump service queuing framework with vacations is useful to involve the idle time for some helpful internal work. A detailed study on vacation queuing models found in [6]. Bunch service queuing framework with Bernoulli vacation has studied by [7]. [8] have discussed bunch arrival retrial queuing framework with modified vacation and N-policy. Retrial queuing framework with Bernoulli vacation using valuable variable strategy studied by a few specialists like [9] and they zeroed in on just single service. Numerous analysts have zeroed in on broad mass service rule which was begun by [10]. [11] Have studied mass queuing model with multiple working vacations. As of late, [12] have discussed cost analysis for group service retrial queuing framework.

Model

The motivation of the proposed queuing framework comes from the exhibition evaluation of neighborhood like carrier sense multiple entrances. The messages divided into bundles. The CSMA can transmit upto B parcels all at once. That is, the server will transmit B parcels in a clump when the quantity of bundles in the orbit is more prominent than or equivalent to B. If lesser then B parcels are in the orbit, then the server will transmit entire bundles in a group. Prior to transmission of the information (bundles), the server checks whether the transmission medium is free or occupied. If the transmission medium is free, then the server will transmit the information. If the CSMA/CA is occupied with service or on another work, arriving bundles need to wait in the orbit (unsatisfied parcels). The parcels from the orbit will retry its allure for service after some arbitrary time. After each transmission of the information, the server might get away or resumes the service. After transmission of the information, if there are no bundles in the orbit the server will go for an optional work. After an optional work, if there are no bundles in the orbit the server needs to wait for new parcels to arrive.

Arrival process

Packets arriving for service according to Poisson process with the rate λ. Where G is the batch size random variable with probability mass function $P\{G = i\} = p_i$, $i = 1, 2, 3\ldots$, probability generating function $G(z) = \sum_{i=0}^{\infty} p_i z^i$ and mean batch size $E(G)$.

Service process

The server will transmit the bundles according to the variable server capacity mass service rule. The variable server capacity group service decide states that the server provides service either fixed size, say 'B' or entire clients from the orbit whichever is lower. After transmission of the gathering of parcels, if the quantity of parcels in the orbit is more noteworthy than or equivalent to 'B', then, at that point, the server will take to transmit 'B' bundles in a cluster. After transmission the parcels, if lesser than 'B' bundles are in the orbit, then, at that point, the server will take entire bundles to transmit in a clump. When service began, late arrivals are not permitted to join in the ongoing service even the size of the clump is lesser than 'B'.

Retrial process

If arriving parcels not getting service immediately because of certain reasons. They are gathered and it named as orbit or unsatisfied clients. Unsatisfied parcels will retry for service after some irregular time.

Vacation process

After completion of an each bunch of transmission, if the orbit is non-unfilled, then the server might decide to go for a vacation with probability p or resumes its service for next group with probability p. After completion of a bunch of transmission, if the orbit is unfilled, then, at that point, the server will go for a vacation. On completion of a vacation, if the orbit is non-unfilled, then, at that point, the server will begin to provide service. On completion of a vacation, if the orbit is unfilled, then, at that point, the server will wait for new clients to arrive.

Notations

We define the probability densities as follows

$$I_0(t) = P\{Z(t) = 0, N_q(t) = 0\}$$

$$I_n(x,t)dx = P\{Z(t) = 0, N_q(t) = n, x < \Theta_0(t) \le x + dx\}, \quad x \ge 0, \quad n \ge 1$$

$$S_n(x,t)dx = P\{Z(t) = 1, N_q(t) = n, x < \Theta_1(t) \le x + dx\}, \quad x \ge 0, n \ge 1$$

$$V_n(x,t)dx = P\{Z(t) = 2, N_q(t) = n, x < \Theta_2(t) \le x + dx\}, \quad x \ge 0, n \ge 0$$

Let $\delta(x)$, $\mu(x)$, $\eta(x)$ be the conditional completion rates for retrial, service and vacation times.

$$\delta(x) = \frac{r(x)}{1 - R(x)}, \quad \mu(x) = \frac{b(x)}{1 - B(x)}, \quad \eta(x) = \frac{c(x)}{1 - C(x)}$$

where

Steady state system equations

$$\lambda I_0 = \int_0^\infty V_0(x)\eta(x)dx \tag{1}$$

$$\frac{dI_n(x)}{dx} = -(\lambda + \delta(x))\, I_n(x) \tag{2}$$

$$\frac{dS_0(x)}{dx} = -(\lambda + \mu(x))\, S_0(x) \tag{3}$$

$$\frac{dS_n(x)}{dx} = -(\lambda + \mu(x))\, S_n(x) + \lambda \sum_{k=1}^{n} g_k S_{n-k}(x) \tag{4}$$

$$\frac{dV_0(x)}{dx} = -(\lambda + \eta(x))\, V_0(x) \tag{5}$$

$$\frac{dV_n(x)}{dx} = -(\lambda + \eta(x))\, V_n(x) + \lambda \sum_{k=1}^{n} g_k V_{n-k}(x) \tag{6}$$

the boundary conditions are

$$I_n(0) = \overline{p}\int_0^\infty S_n(x)\mu(x)dx + \int_0^\infty V_n(x)\eta(x)dx \tag{7}$$

$$S_0(0) = \int_0^\infty \sum_{n=1}^{B} I_n(x)\delta(x)dx + \lambda \sum_{k=1}^{B} g_k I_0 \tag{8}$$

$$S_n(0) = \int_0^\infty I_{n+B}(x)\delta(x)dx + \lambda \int_0^\infty \sum_{k=1}^{n} g_k I_{n-k+B}(x)dx + \lambda g_{n+B} I_0 \tag{9}$$

$$V_n(0) = p\int_0^\infty S_n(x)\mu(x)dx \tag{10}$$

$$V_0(0) = p\int_0^\infty S_0(x)\mu(x)dx \tag{11}$$

and the normalisation condition is

$$I_0 + \sum_{n=1}^{\infty}\int_0^{\infty} I_n(x)dx + \sum_{n=0}^{\infty}\int_0^{\infty} S_n(x)dx + \sum_{n=0}^{\infty}\int_0^{\infty} V_n(x)dx = 1 \qquad (12)$$

To solve the equations (1) to (11), we are defining the following generating functions

$$I(x,z) = \sum_{n=1}^{\infty} I_n(x)z^n \quad S(x,z) = \sum_{n=0}^{\infty} S_n(x)z^n \quad S(x,z) = \sum_{n=0}^{\infty} S_n(x)z^n$$

Multiplying equations (1 to (11) by z^n and taking summation over n; n = 1,2,3…, we attained the following equations:

$$\frac{\partial I(x,z)}{\partial x} + (\lambda + \delta(x))I(x,z) = 0 \qquad (13)$$

$$\frac{\partial S(x,z)}{\partial x} + (\lambda + \mu(x) - \lambda G(z))S(x,z) = 0 \qquad (14)$$

$$\frac{\partial V(x,z)}{\partial x} + (\lambda + \eta(x) - \lambda G(z))V(x,z) = 0 \qquad (15)$$

$$I(0,Z) = \overline{p}\int_0^{\infty} S(x,z)\mu(x)dx + \int_0^{\infty} V(x,z)\eta(x)dx - \lambda I_0 - \overline{p}V_0(0) \qquad (16)$$

$$S(0,z) = \int_0^{\infty}\frac{I(x,z)}{z^B}\delta(x)dx + \frac{\lambda G(z)}{z^B}I_0 + \frac{\lambda G(x)}{z^B}\int_0^{\infty} I(x,z)dx \qquad (17)$$

$$V(0,z) = p\int_0^{\infty} S(x,z)\mu(x)dx + \overline{p}V_0(0) \qquad (18)$$

By solving the differential-difference equations (13) to (15), we obtained as follows

$$I(x,z) = I(0,z)e^{-\lambda x - \int_0^x \delta(x)dx} \qquad (19)$$

$$S(x,z) = S(0,z)e^{-\lambda(1-G(z))x - \int_0^x \mu(x)dx} \qquad (20)$$

$$V(x,z) = V(0,z)e^{-\lambda(1-G(z))x - \int_0^x \eta(x)dx} \qquad (21)$$

From equation (5), we get

From equation (5), we get

$$V_0(0) = \frac{\lambda I_0}{\gamma^*(\lambda)} \qquad (22)$$

Using equations (20) to (22) in the equation (16), we get

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$$I(0,z)=\frac{I_0}{\gamma^*(\lambda)}\left(\frac{\dfrac{\lambda\gamma^*(\lambda)S^*(\lambda-\lambda G(z))G(z)\left[\overline{p}+p\gamma^*(\lambda-\lambda G(z))\right]+\lambda z^B\left[\overline{p}\left(\gamma^*(\lambda-\lambda G(z))-1\right)-\gamma^*(\lambda)\right]}{z^B-S^*(\lambda-\lambda G(z))}}{\left[\overline{p}+p\gamma^*(\lambda-\lambda G(z))\right]\left[\alpha^*(\lambda)+G(z)(1-\alpha^*(\lambda))\right]}\right) \tag{23}$$

Using equation (19) in the equation (17), we get

$$S(0,z)=\frac{I_0}{\gamma^*(\lambda)}\left(\frac{\dfrac{\lambda\overline{p}\left(\gamma^*(\lambda-\lambda G(z))-1\right)\left[\alpha^*(\lambda)+G(z)(1-\alpha^*(\lambda))\right]+\lambda\alpha^*(\lambda)\gamma^*(\lambda)(G(z)-1)}{z^B-S^*(\lambda-\lambda G(z))}}{\left[\overline{p}+p\gamma^*(\lambda-\lambda G(z))\right]\left[\alpha^*(\lambda)+G(z)(1-\alpha^*(\lambda))\right]}\right) \tag{24}$$

Using equation (20) and (22) in the equation (18), we get

$$V(0,z)=\frac{I_0}{\gamma^*(\lambda)}\left(\frac{\dfrac{\lambda\overline{p}\left[z^B-S^*(\lambda-\lambda G(z))\left(\alpha^*(\lambda)+G(z)(1-\alpha^*(\lambda))\right)\right]+\lambda p\alpha^*(\lambda)\gamma^*(\lambda)S^*(\lambda-\lambda G(z))(G(z)-1)}{z^B-S^*(\lambda-\lambda G(z))}}{\left[\overline{p}+p\gamma^*(\lambda-\lambda G(z))\right]\left[\alpha^*(\lambda)+G(z)(1-\alpha^*(\lambda))\right]}\right) \tag{25}$$

Conclusion

There are numerous extensive works have been finished in the vacation model region throughout the course of recent a long time as studied in [13], [14], [15] and some overview papers [16] [17] including this paper. In this work, we presented short overview of the studies on variants of arrival cycle and service processes in working vacation queuing models. The idea discussed in various papers has been synthesized. It can help statisticians, operations investigator, scientists, engineers, directors for using these models. A wide scope of literature has been covered and legitimate references have been cited.

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